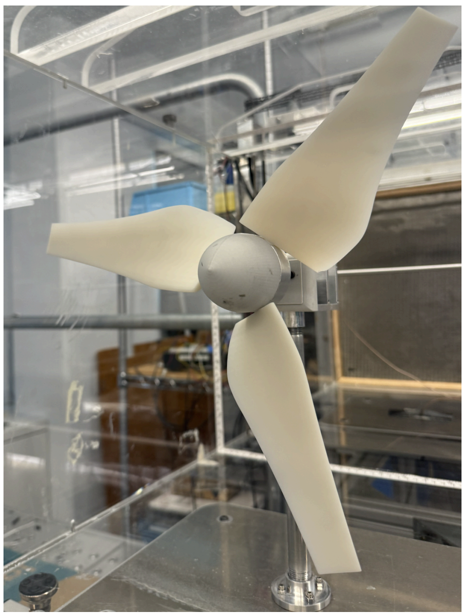
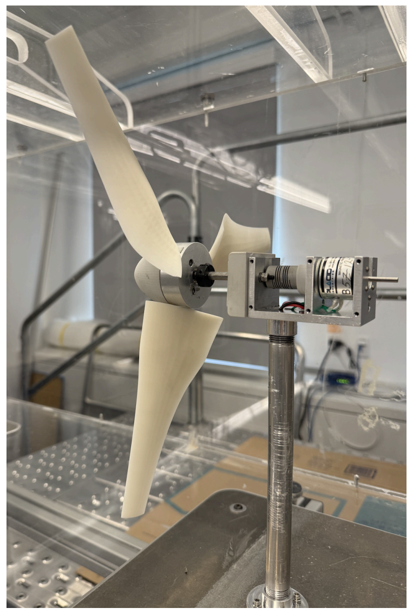
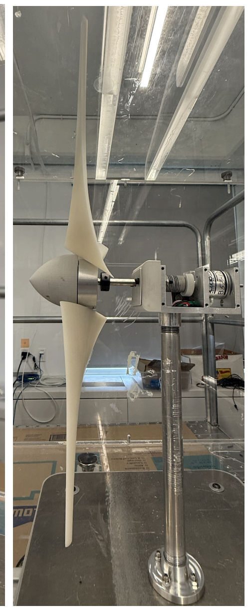
MAE 4272 – Fluids and Heat Transfer Laboratory

Small wind-turbine blades design project – Final Report

All Swag, No Drag

Group 404-3

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1. Executive Summary

Our project focuses on designing a three blade wind turbine for the MAE 4272 Big Blue wind tunnel that maximizes power output while remaining structurally safe and compatible with the experimental and physical constraints. We combined theoretical principles such as the Weibull distribution for wind characterization, 1-D Momentum theory, and Blade Element-Momentum theory to design an efficient blade targeting a specific tip speed ratio. Using a MATLAB script, we generated and evaluated ideal chord and pitch distributions across a range of angular velocities and wind speeds. The script incorporated low-Reynolds NACA 66(1)-212 polar data, accounted for axial wind velocity reduction induced by the foils, matched the local tip-speed-ratio distribution along the span, and swept angles of attack within the pre-stall range. The result was a high-resolution 3D blade geometry generated by repeatedly computing the spanwise lift and drag tangential forces and iterating until converging on the maximum torque. For our geometry, we also evaluated the bending stress distribution along the blade to ensure the design remained structurally safe under the expected operating conditions.

After selecting the final design and modeling and manufacturing the blades, we experimentally developed power curves for our turbine at a range of frequencies to characterize its performance at various wind speeds. The rotor achieved its predicted maximum torque at the intended operating wind speed and even demonstrated stronger performance than expected. It was able to continue spinning under the maximum allowable brake voltage, clearly showing its ability to produce more power and highlighting the strength of the optimization approach and aerodynamic model.

2. System Context and Constraints

a. Physical Constraints

Since the turbine is meant to function in our lab wind tunnel, it has to follow specific constraints. Each blade must remain under the 6 inch span limit. The blade root must be compatible with the 1-inch hub. To ensure safe operation the turbine cannot exceed 3000 RPM. All blades were grey resin 3D prints with a flexural strength of 44 MPa. Our required FOS was 1.5 to ensure there are no failures.

b. Wind Operating Context

The turbine operates in the Big Blue wind tunnel where the wind speed is driven by a variable frequency fan. To understand the wind conditions we should design to, we used the Weibull probability density function to understand the wind velocity distribution.

$$p(U) = (k/c)\,(U/c)^{(k-1)}\,exp[-\,(U/c)^{k}]$$

From our analysis, we determined that our mean wind speed is 4.782 m/s with a standard deviation of ±1.052 m/s. These wind speeds directly informed our design choices as we used them to establish our operating point and rated rotational speed. Incorporating this wind speed distribution into our design ensured that the blades not only were not just tuned to be most efficient at one speed, but also robust at a reasonable range of the tunnel's airflow.

3. Design Process

a. Theoretical Foundation

In order to design an efficient wind turbine, it's important to understand how a rotor extracts energy from the wind and how it relates to operating conditions and blade geometry. We used 1-D momentum theory to define the axial induction factor a which is how much freestream velocity U is reduced at the rotor plane. The velocity at the disk is defined by:

$$U_d = U_\infty(1 - a)$$

The reduction of velocity is crucial in generating power as it does this by slowing the wind speed down and converting the loss of kinetic energy to useful torque. Downstream of the rotor, the wake velocity is expressed by

$$U_w = U_\infty(1 - 2a)$$

Using a momentum balance, the mass flow through the disk is therefore

$$\dot{m} = \rho A U_d = \rho A U_\infty(1 - a)$$

Thrust derived from the change in momentum:

$$T = \dot{m}(U_\infty - U_W) = 2\rho A U_\infty^{2} a(1 - a)$$

Multiplying this by the local velocity (U_d), we obtain the power extracted at disk

$$P = TU_d = 2\rho A U_\infty^{3}\ a(1 - a)^2$$

To determine wind turbine efficiency, we define the total power available in the wind as

$$P_{avail} = 1/2\ \rho A U_\infty^{3}$$

The ratio of the extracted power to the available power is known as the power coefficient:

$$C_P = \frac{P}{P_{avail}} = \frac{2\rho A U_\infty^{3} a(1-a)^2}{\frac{1}{2}\rho A U_\infty^{3}} = 4a(1 - a)^2$$

The main goal is to maximize C_P for maximum efficiency so:

$$\frac{d}{da}\{\,[4a(1 - a)^2] = 4(1 - 4a + 3a^2)\,\} = 0 \ \Rightarrow a = 1/3$$

Substituting a=⅓ to the power coefficient equation, we get the Betz limit of 16/27 (.593) which is the maximum theoretical efficiency of a turbine. We take $a = 1/3$ to simplify our analysis, but still produce physically reasonable estimates.

We additionally apply Blade Element Momentum (BEM) to derive an optimal chord distribution for each blade on the turbine. BEM combines the conservation of momentum requirement with blade aerodynamics, providing a direct relationship between flow conditions, airfoil properties and blade geometry needed for a high efficiency. The optimal chord distribution (as a function of r) is described by

$$c(r) = \frac{8\pi r}{BC_L}(1 - cos\phi)$$

To solve for this distribution, we find the local inflow angles, $\phi(r)$

$$tan\phi = \frac{1-\alpha}{\lambda_r}$$

$$\text{Where } \lambda_r = \frac{\Omega r}{U}$$

Where B=3 to account for the number of turbine blades under test, Ω is the turbine angular velocity, α is the angle of attack of the airfoil cross-section at radius r, and C_L is given by the tabulated polar for our chosen airfoil (NACA 66(1)-212).

b. Tip Speed Ratio and Rated Speed

Tip speed ratio is defined as $\lambda = \Omega R / U$, where Ω is the rotational speed of the rotor is rad/s, R is the rotor radius, and U is the free stream wind velocity. It relates the blade tip speed to the wind speed. This ratio matters because it governs all aerodynamic behavior of a wind turbine and with a properly chosen TSR we can achieve maximum power according to BEM theory. We are trying to achieve a TSR of 6 because this is a parameter that has been proven to optimize wind power with peak aerodynamic efficiency.

c. Airfoil Selection

We selected the NACA 66(1)-212 because it shares the same fundamental geometry as the NACA 4412 and NACA 0012 airfoils used in lab, which had already demonstrated structural reliability and strong power performance. From airfoiltools, we found that the NACA 66(1)-212 offered a slightly higher C_L/C_D ratio at our expected operating angle of attack. In theory, this allowed us to extract more power without sacrificing structural integrity, while minimizing the risk of unexpected behavior due to its similarity to the lab airfoils we have worked with.

d. MATLAB Optimization

The optimization script searches over rotor speed and angle of attack to find the blade geometry that produces the most predicted power, according to the following algorithm:

input: span **r,** angles of attack **α,** angular speeds **Ω**, low-Re Xfoil data $[\mathbf{C_L}, \mathbf{C_D}]$

for each Ω_i:

for each α_j:

$$\phi(r) = atan(\frac{U(1-\alpha_{ind})}{\Omega r})$$

$$\beta(r) = \phi(r) - \alpha$$

$$c(r) = f(\Omega_i , r , U_{wind} , [C_L ,C_D])$$

$$F_{useful} = f(\ [C_L, C_D]\ , c(r)\ , \beta(r)_j\ )$$

$$P_{\Omega i,\beta j} = F_{useful} \cdot \Omega_i$$

return $[\beta(r), c(r), \Omega] = \mathrm{argmax}(\ P_{\Omega i,\beta j}\ )$

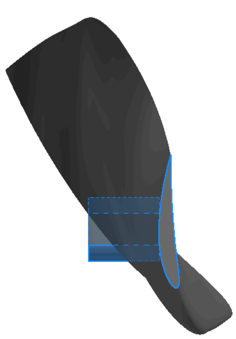
It loads an XFOIL airfoil polar containing, $C_L(\alpha)$, $C_D(\alpha)$, sets the environment and rotor size (3 blades, a 1-inch hub cutout, 6-inch aerodynamic span), and discretizes the blade from root to tip into 40 radial stations. It then defines a sweep of angles of attack from 6 to 12 degrees and a sweep of angular velocities Ω chosen about a target tip-speed rato $\lambda = 6$.

For each (Ω, α) pair, it computes the spanwise inflow angle $\phi(r)$ using the simple Glauert inflow model with a fixed axial induction factor a = ⅓, as described above, then sets the required pitch distribution $\beta(r) = \phi(r) - \alpha$. It looks up C_L and C_D from the polar (with clamps to keep values in a realistic pre-stall range), computes an optimal chord distribution c(r) using the Betz/Glauert chord formula described above with practical caps, and then estimates the useful tangential aerodynamic force at each radial station. Integrating that force over the span gives torque, scalar power is computed as $P = \Omega T$, and the script selects the maximum-power case and exports the corresponding chord $c(r)$ and pitch $\beta(r)$ distributions to a CSV.

This blade geometry and optimized aerodynamic loads were then fed into a bending‑stress model that calculated bending stress along the foil's radius, giving stresses on the order of 0.24 MPa for our expected wind conditions and a maximum of 9 MPa even at much higher speeds (60 m/s); the stresses were well below the 58 MPa flexural strength of Accura 25. However, these stresses are likely underestimated because the model is highly simplified, assuming static normal-to-chord bending, neglecting print defects, stress concentrations, and dynamic effects like aeroelastic flutter. As a result, we proceeded with caution throughout testing.

e. CAD Modeling

The blade was lofted in Fusion 360 using interpolated cross sections of the NACA 66(1)-212 along with the optimized chord and twist profiles.



4. Experimental Procedure

Although our turbine is designed to be optimal at a specific wind speed we decided to test the turbine at different wind speeds to get a full picture because we were not sure if our design would operate as we planned. First we zeroed the pressure transducer and then tried to pick a starting fan speed. At low fan speeds (< 3Hz), our turbine didn't spin so we decided to increase the fan speed until our turbine started to spin, then decrease the frequency to test at these lower speeds. At each fan speed we incremented the torque brake voltage by .4V then waited until RPM was stable meaning the turbine was in equilibrium. Then we pulled data and kept increasing the torque brake until the turbine either stalled (indicating that maximum power was reached) or we reached the torque brake voltage limit. We repeated this process for all fan speeds. With the data we collected we are able to analyze our blade at different operating conditions and draw conclusions about our blade performance using power curves and efficiency calculations.

5. Results

a. Satisfied Blade Constraints

The physical printed airfoil satisfied all defined constraints. It properly interfaced with the hub and was within the 6 inch maximum span. Furthermore, it did not face any structural damage despite increased displacement during vibrational nodes and was able to spin at relatively high RPMs within all speeds given by the wind speed distribution.

b. Power Curves

We developed power curves for our turbine at 4, 6, 8, and 10 Hz. At 4 Hz and 6 Hz (2.18 m/s and 3.17 m/s), the turbine did stall before maxing out the torque brake voltage (10 V) meaning that we could extract a maximum power value from those trials. However our 8 Hz and 10 Hz tests (4.33 m/s and 5.394 m/s) did not stall before the brake limit; it kept spinning reflecting the ability to produce more power and thus we were unable to determine the actual maximum power that could have been extracted.

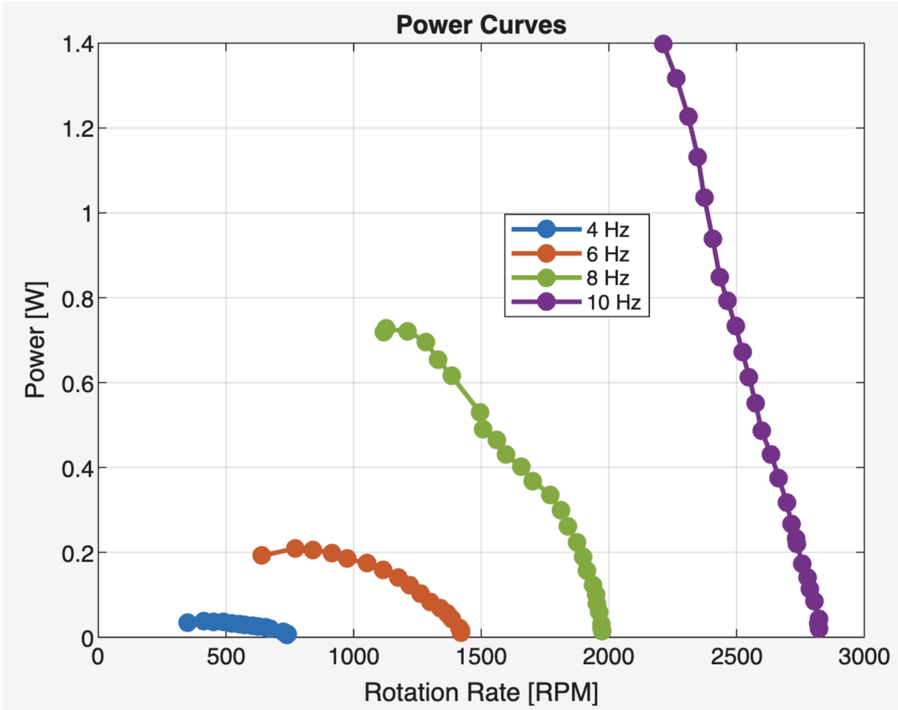


Figure 1: Power Curves

Figure 1 shows the power curves from four trials overlaid on one plot. The 8 Hz and 10 Hz trials produced 0.73 W at 1126 RPM and 1.4 W at 2213 RPM, respectively, at a 9.6 V brake max setting. This demonstrates the potential for increased power at higher applied brake torque. While power rose with wind speed (increased wind speed meant increased RPM), the maximum torque of 0.0062 Nm occurred at the wind speed closest to our optimized MATLAB design, confirming the effectiveness of the optimizer.

c. Efficiency

While power may increase with wind speed, that does not necessarily mean that efficiency also peaks at maximum power. We actually found that our most efficient experiment was at our 8Hz (4.33 m/s) fan and wind speed. The torque and efficiency peak near the designed operating power confirms that the turbine geometry was well tuned to our target wind distribution. Below is an example calculation of efficiency using data from our 8Hz trial:

$$C_P = \frac{P}{P_{avail}} = \frac{P}{\frac{1}{2}\rho A U_\infty^{\ 3}} = \frac{.73}{\frac{1}{2}(1.2)(7.30E-2)(4.33)^3}$$

$$= 21\%$$

Here is a summary table of our results:

Test Fan Frequency (Hz)	**4**	**6**	**8**	**10**
Wind Speed (m/s)	2.18	3.17	4.33	5.395
Max Power (W)	.037	.21	.73	1.4
Rotation Rate (RPM) at Max Power	412	770	1126	2213
Torque (N-m) at Max Power	.00087	.0026	.0062	.006
Efficiency (%)	8	15	21	20

Table 1: Results Summary

d. Next Iteration

For a second iteration, there are a few improvements to the test setup and blade that could be made. We would retune the tower height to achieve anti-resonance at the peak power RPM and reduce vibratory loads. Instrumentation upgrades would include a higher-torque brake and a more rigid hub-blade connection to improve torque measurement fidelity. Lastly, CAD refinements will use a higher-resolution loft and smoother blade–hub transitions and tip geometry to minimize parasitic drag.

6. Conclusion

Our final design met the constraints and achieved strong aerodynamic performance. We were able to design our turbine to have the maximum efficiency corresponding to the wind speed we designed to be determined by the lower end of the Weibull distribution mean wind speed. The turbine had stable behavior, high efficiency, and no structural problems at all operating conditions. In terms of improvement, in order to extract maximum power we would switch out the torque brake for a stronger one, or we would focus more on staying within the torque limits of the torque brake.

Overall, our team worked well together. We worked effectively by dividing tasks according to our strengths and communicating our progress. We had separate members each taking on different roles such as CAD, understanding theory, blade design matlab script, structural analysis, results analysis, and power curve generation. On parts we were unsure about, we would meet together and all discuss working through our approach and any problems. There were a lot of parts to this project but our group was able to tackle them all on time, and do it well, so it was a rewarding experience for everyone involved.